A practical calculating model including multi-mode contributions for along-wind responses of lattice towers

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Abstract:

A practical calculating model, based on the fundamental mode generalized force spectrum (FMGFS) obtained in a wind tunnel test and presented practical higher mode generalized force spectrum (HMGFS) model in along-wind direction of lattice tower, is further deduced and proposed to calculate along-wind displacement response of lattice tower. In the proposed model, the contributions of higher vibration modes can be taken into account. As for lattice tower, it is of value to popularize the proposed model and the approach that can provide valuable reference for code.

Keywords: *Along-wind-induced dynamic responses; Transmission tower; Generalized force spectrum*.

1. Introduction

Lattice high-rise structure such as transmission tower, with the characters of lightweight, highness, flexibility and low damping, is sensitive to wind load[1]. It is a necessary condition to accurately calculate the wind load before further conducting the structural design. The wind load on structure can be qualified through the multiple-point synchronous scanning of pressures (MPSSP) on a structure model surface in a wind tunnel, or by a high frequency fore balance (HFFB) measure. Different from the ordinary building, however, the wind load on the transmission tower is difficult to, even can not, be measured in detail using MPSSP measures due to its high hollowness rate, while the HFFB measure is usually used to offer an estimate of the generalized force of the fundamental mode and the FMGFS. Then, according to the random vibration theory, the variance of the responses only including the first mode can be further deduced. However, higher modes may have noticeable contributions to the responses, especially for those slender structures like transmission tower. A systematic work to evaluate along-wind responses of latticed tower was studied by Holmes[2,3,4]. In order to have better understanding of transmission tower-lines system to wind excitation, a novel approach that for the wind tunnel aeroelastic modelling of conductors was introduced in detail by Loredo-Souza[5,6]. More discussions concerning with the generalized force spectrum of transmission tower have not been addressed. The studies in which three semi-rid tower models were made and HFFB measures were used in TJ-1 boundary layer wind tunnel in Tongji University firstly obtained the FMGFS of transmission tower, and then the HMGFS in an analytic form was also discussed [7]. In order to obtain structural internal force, like the conventional processing approach, the ESWLs only including fundamental modal contribution of transmission tower was also presented [8]. In this paper, for the purpose of practical engineering applications, a practical numerical HMGFS model is firstly presented on the basis of the fundamental mode generalized force spectrum (FMGFS) obtained from a wind tunnel model experiment and adopting the height-independent fluctuating wind power spectral density and Shiotami-Type spatial coherence function. Then, a practical calculating model for the displacement response is further deduced. In the formula, the contributions of higher modes can be included and the conversion relations between the unilateral and bilateral power spectral densities are

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also taken into account in the derivation. er structural design.

2. Practical HMGFS model

For a structural system with rigid-bases, its dynamic equilibrium equations under the wind load action can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t)+\boldsymbol{C}\dot{\boldsymbol{y}}(t)+\boldsymbol{K}\boldsymbol{y}(t)=\boldsymbol{F}(z,t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ denote the mass, damping and stiffness matrices, $\boldsymbol{y}$(t), $\dot{\boldsymbol{y}}$ (t) and $\ddot{\boldsymbol{y}}$ (t) mean the displacement, velocity and acceleration vectors, respectively, $\boldsymbol{F}$(z, t) implies the nodal force vector. Then, the typical uncoupled modal equation for the structural system is of the following form:

$$M_n\left[\ddot{q}_n(t)+2\xi_n\omega_n\dot{q}_n(t)+\omega_n^2 q_n(t)\right]=F_n^G(z,t) \tag{2}$$

in which

$$M_n=\boldsymbol{\Phi}_n^T\boldsymbol{M}\boldsymbol{\Phi}_n$$

$$F_n^G(z,t)=\boldsymbol{\Phi}_n^T\boldsymbol{F}(z,t)$$

where $\boldsymbol{M}_n$ is the generalized mass. $\boldsymbol{\Phi}_n$ and q_n(t) are the modal and generalized displacement vectors, ξ_n and ω_n represent the damping ratio and circular frequency, respectively, F_n(z, t) means the generalized force, here the subscript (n) refers to the *n*th mode. According to the theory of quasi-stability, the *n*th modal generalized force is expressed as follows

$$F_n^G(z,t)=\frac{1}{2}\rho C_D\boldsymbol{\Phi}_n^T\left\{B(z)d(z)R(z)U(z,t)^2\right\} \tag{3}$$

where ρ denotes the air density, C_d means the air damping coefficient, B(z) denotes the calculating width and d(z) implies the calculation height at al titude, z, and R(z) refers to the ratio between act ual area of unit height and contour area and

$$U(z,t)^2=\bar{U}(z)^2+2\bar{U}(z)\tilde{U}(z)+\tilde{U}(z)^2 \tag{4}$$

where $\bar{U}$ (z) and $\tilde{U}$ (z) are the mean wind speed and the fluctuating wind speed. Here, the effect of $\bar{U}$ (z) is equivalent to static force. Actually, compared with $\bar{U}$ (z)^2 and 2$\bar{U}$ (z)$\tilde{U}$ (z), $\tilde{U}$ (z)^2 is very small and can be neglected[9]. Thus, substituting Eq. (4) into Eq. (3) yields

$$F_n^G(z,t)=\rho C_D\boldsymbol{\Phi}_n^T\left\{\Gamma(z,t)\right\} \tag{5}$$

in which

$$\Gamma(z,t)=B(z)d(z)R(z)\bar{U}(z)\tilde{U}(z,t)=\kappa(z)\tilde{U}(z,t) \tag{6}$$

and

$$\kappa(z)=B(z)d(z)R(z)\bar{U}(z)\tilde{U}(z,t) \tag{7}$$

Strictly speaking, it is more reasonable that fluctuating wind speed spectrum density should vary with the height, z. In accordance with the criterion of engineering applications, however, the influence of z on the structural responses is very small and can be neglected [10]. Hence, according to the random vibration theory and matrix theory, the power spectrum of the *n*th generalized force can be expressed as

$$S_n^G(f)=\rho^2C_D^2\boldsymbol{\Phi}_n^T\boldsymbol{\Lambda}\boldsymbol{\Phi}_n \tag{8}$$

where

$$\boldsymbol{\Lambda}=\begin{bmatrix}\Delta(z_1,z_1,f) & \cdots & \Delta(z_1,z_i,f) & \cdots & \Delta(z_1,z_H,f)\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \Delta(z_i,z_1,f) & \cdots & \Delta(z_i,z_i,f) & \cdots & \Delta(z_i,z_H,f)\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \Delta(z_H,z_1,f) & \cdots & \Delta(z_H,z_i,f) & \cdots & \Delta(z_H,z_H,f)\end{bmatrix}S(f) \tag{9}$$

$$\Delta(z_i,z_j,f)=\rho(z_i,z_j,f)\kappa(z_\mathrm{i})\kappa(z_\mathrm{j}) \tag{10}$$

where S(f) represets the fluctuating wind speed spectrum density which is independent of z and ρ(z_i, z_j, f) is the correlation coefficient between z_i and z_j. In theory, the correlation coefficient is a multivariant function, not only related to the position, but also the frequency. Here, the Eq.(11) suggested by Shiotami is adopted because of the following reasons: Eq.(11) was established on the basis of test and is applicable to high-rise structures[10], and it is quite convenient to be used due to that it has nothing to do with frequency:

$$\rho(z_i,z_j)=\exp\left(-\frac{\left|z_i-z_j\right|}{L_z}\right),\ L_z=60 \tag{11}$$

Substituting Eqs(9), (10) and (11) into Eq. (8) gives

$$S_n^G(f)=\rho^2 C_D^2 \boldsymbol{\varphi}_n^T \boldsymbol{L}_R \boldsymbol{\varphi}_n S(f)\,,\,(n\ge 1, n\in Z\,) \quad (12)$$

where $\boldsymbol{L}_R$ is defined as the location correlation matrix, the element of which is expressed as follows

$$L_R(i,j)=\rho(z_i\,,z_j)\kappa(z_i)\kappa(z_j) \quad (13)$$

From Eq.(12), the HMGFS expression can be obtained and expressed as

$$S_n^G(f)=\frac{\boldsymbol{\Phi}_n^T \boldsymbol{L}_R \boldsymbol{\Phi}_n}{\boldsymbol{\Phi}_1^T \boldsymbol{L}_R \boldsymbol{\Phi}_n} S_1^G(f)=\Theta_n S_1^G(f) \quad (n\ge 1) \quad (14)$$

in which

$$S_1^G(f)=\frac{a(fB/V_H)^{1.05}}{f[1+b(fB/V_H)^2]^{1.5}}\sigma_1^2 \quad (15)$$

$$\sigma_1=\frac{1}{2}C_M\rho V_H^2 AR=\sqrt{\int_0^\infty S_1^G(f)df} \quad (16)$$

where Θ_n is defined as the generalized force spectral mode coefficient, $S_1^G(f)$ referes to the FMGFS, A, B and σ_1 mean the contour area, bottom-width of transmission in the direction of approach flow and root mean square(RMS) of the fundamental mode generalized force, a, b and C_M are the fitting parameters, which are respectively 14.6, 113.5 and 0.088 in turn[7]. From Eq.(14), it can be found that the Θ_n establishes the constitutive relation between $S_n^G(f)$ and $S_1^G(f)$, which conforms to physical meaning. Compared with the analytical model in Eq.(17), Eq.(14) is more practical and easy to be adopted by researchers or civil engineers.

$$S_n^G(f)=\frac{\int_0^H\int_0^H \Omega_1\Omega_2\rho(z_i,z_j)\phi_n(z_1)\phi_n(z_2)dz_1dz_2}{\int_0^H\int_0^H \Omega_1\Omega_2\rho(z_i,z_j)\frac{z_1}{H}\frac{z_2}{H}dz_1dz_2}S_1^G(f) \quad (17)$$

$$\Omega_1=\frac{1}{2}\rho C_D S(z_1)\bar{V}(z_1)^2 \quad (18)$$

$$\Omega_2=\frac{1}{2}\rho C_D S(z_2)\bar{V}(z_2)^2 \quad (19)$$

in which $S(z)$ denotes the solid area at the height, z.

3. Multi-modal model for displacement response

It is the purpose of this section, based on the FMGFS obtained from the wind-tunnel and the HMGFS deduced in section 2.1, to give a practical calculation formula to evaluate along-wind displacement response of the structure.

According to modal superposition method[11], the displacements of lattice tower at different heights can be given by

$$y(z,t)=\sum_{n=1}^N \phi_n(z)q_n(t) \quad (20)$$

Accordingly, based on random vibration theory, the transfer function can be written in the following form

$$H_y(z,i\omega)=\sum_{n=1}^N \phi_n(z)H_n(i\omega) \quad (21)$$

$$H_n(i\omega)=\left\{K_n\left[1+i2\xi(\omega/\omega_n)-(\omega/\omega_n)^2\right]\right\}^{-1} \quad (22)$$

where $q_n(t)$ is the nth modal generalized displacement and $K_n=M_n\omega_n^2$ denotes the generalized rigidity of the nth mode. It is apparent that the physical phenomenon of structural vibration subjected to the wind load may be looked upon as a multiple-input-multiple-output random process. Hence, the output power spectrum for the displacement can be written as follows

$$S_y(z,\omega)=\sum_{n=1}^N \phi_n^2(z)\left|H_n(i\omega)\right|^2 R_n(\omega) + \sum_{\substack{k=1\\k\ne l}}^N\sum_{l=1}^N \phi_k(z)\phi_l(z)H_k(i\omega)H_l(i\omega)\gamma_{kl}\sqrt{R_k(\omega)R_l(\omega)} \quad (23)$$

where $R_{j(k,l)}(\omega)$ is the cross-power spectral density of the j(k,l)th mode, which is the bilateral power spectrum in the strict mathematical sense and will be replaced by the corresponding unilateral generalized force spectrum $S_n^G(f)$ mentioned above, γ_{kl} represents the correlation coefficient between the mth and nth mode and not only dependent on the frequency ratio and damping ratio, but also the coherence of generalized force spectrum，and can be estimated as follows[12,13,14]:

$$\gamma_{kl}=\rho_{kl}\iota_{kl} \quad (24)$$

in which

$$\rho_{kl}=\frac{8\sqrt{\xi_l\xi_k}\left(\lambda_{kl}\xi_l+\xi_l\right)\lambda_{kl}^{3/2}}{\left(1-\lambda_{kl}^2\right)^2+4\xi_k\xi_l\lambda_{kl}\left(1+\lambda_{kl}^2\right)+4\left(\xi_k^2+\xi_l^2\right)\lambda_{kl}^2} \tag{25}$$

$$\iota_{kl}=\mathrm{Re}\left[R_{kl}(\omega)\right]/\sqrt{R_k(\omega)R_k(\omega)}\Big|_{\omega=\omega_m or \omega_l} \tag{26}$$

where $0<\lambda_{mn}=f_m/f_n<1$ and Re denotes the real part of the corresponding complex value.

Taking into account small damping and discrete natural frequencies of the lattice tower, the cross-terms in Eq.(23) can be neglected. Then, the root-mean-square (RMS) value for the displacement response is given by

$$\begin{aligned}\sigma_y(z)&=\sqrt{\int_{-\infty}^{+\infty}\mathrm{S}_y(z,\omega)d\omega}\\&=\sqrt{\sum_{n=1}^{N}\phi_n^2(z)\int_{-\infty}^{+\infty}\left|H_n(i\omega)\right|^2R_n(\omega)d\omega}\end{aligned} \tag{27}$$

$$\left|H_n(i\omega)\right|^2=\left\{K_n^2\left[\left(1-(\omega/\omega_n)^2\right)^2+4\xi^2(\omega/\omega_n)^2\right]\right\}^{-1} \tag{28}$$

Phasing is adopted to transfer Eq.(27) into the algebra expression and the full integration can be separated into three integrating ranges: (1) while $|H_n(i\omega)|^2$ is over the resonant frequency range, $\omega_n-\delta/2\le\omega\le\omega_n+\delta/2$, the covibration is significant and the dynamic response becomes prominent; (2) while $\omega<\omega_n-\delta/2$, the response may be approximately considered as the quasi-static due to that $|H_n(i\omega)|^2$ is a narrow-band random process and appears fast decrease because of the low damping values[8,10]; (3) while $\omega>\omega_n+\delta/2$, $R_n(\omega)$ and $|H_n(i\omega)|^2$ exhibit a sharp decreasing tendency to approach zero[8], so it is reasonable to neglect the influence of this integral range on $\sigma_y(z)$. Then, $|H_n(i\omega)|^2$ and $|H_n(i\omega)|^2R_n(\omega)$ can be approximately estimated as

$$\left|H_n^{"}(i\omega)\right|^2\cong\begin{cases}1/(4K_n^2\xi_n^2)=1/(4M_n^2\omega_n^4\xi_n^2), & \omega\in[\omega_n-\delta/2,\omega_n+\delta/2]\\ 1/K_n^2=1/(M_n^2\omega_n^4), & \omega\in(0,\omega_n-\delta/2)\end{cases} \tag{29}$$

$$\left|H_n^{"}(i\omega)\right|^2\mathrm{R}_n(\omega)\cong0,\ \omega\in(\omega+\delta/2,+\infty) \tag{30}$$

Substituting Eqs.(28), (29) and (30) into Eq.(27) and considering dynamic magnification factor $\beta=1/2\xi_n$ under resonance condition yields

$$\begin{aligned}\sigma_y(z)&=\sqrt{\sum_{n=1}^{N}\phi_n^2(z)\left[\left|H_n^{'}(i\omega)\right|^2\int_{-\infty}^{+\infty}R_n(\omega)d\omega+\left(1-\frac{1}{\beta}\right)^2\left|H_n^{"}(i\omega_n)\right|^2R_n(\omega_n)\delta\right]}\\&=\sqrt{\sum_{n=1}^{N}\frac{\phi_n^2(z)}{M_n^2\omega_n^4}\left[\int_{-\infty}^{+\infty}R_n(\omega)d\omega+\frac{(1-2\xi_n)^2}{4\xi_n^2}R_n(\omega_n)\delta\right]}\end{aligned} \tag{31}$$

Here, the repetitive calculation of static response in the range of resonant frequency is avoided, which was not be considered in the literature [8]. However, in practice, the form of Eq.(31) is still not convenient to be adopted directly by researchers or engineers. To be practical, it is valuable to further give the specific algebraic expressions of δ and $\int_{-\infty}^{+\infty}R_n(\omega)d\omega$.

(1) *determine* δ: $|H_n(i\omega)|^2$ is a narrow-band process and $R_n(\omega)$ is approximately regarded as a white noise within the range of δ[15], in which $R_n(\omega)$ is constantly equal to R_{const}. Then, δ is evaluated as

$$2\delta=\frac{\int_{-\infty}^{+\infty}\left|H_n(i\omega)\right|^2R_{const}d\omega}{\left|H_n(i\omega)\right|^2R_{const}}=\frac{\int_{-\infty}^{+\infty}\left|H_n(i\omega)\right|^2d\omega}{\left[1/(4M_n^2\omega_n^4\xi_n^2)\right]}=2\pi\xi_n\omega_n \tag{32}$$

$$\Rightarrow\delta=\pi\xi_n\omega_n \tag{33}$$

(2) *determine* $\int_{-\infty}^{+\infty}R_n(\omega)d\omega$: Consider $R_n(\omega)$ being an even function and substituting Eq.(14) into it gives

$$\begin{aligned}\int_{-\infty}^{+\infty}\mathrm{R}_n(\omega)d\omega&=2\int_0^{\infty}\mathrm{R}_n(\omega)d\omega=2\int_0^{+\infty}\frac{\Theta_nS_1(f)}{4\pi}2\pi df\\&=\Theta_n\int_0^{+\infty}S_1(f)df=\Theta_n\sigma_1^2\end{aligned} \tag{34}$$

It is evident that Θ_n has established the constitutive relation between $\int_{-\infty}^{+\infty}R_n(\omega)d\omega$ and σ_1^2. Substitutions of Eqs.(33) and (34) into eq.(31) finally gives the following algebraic expressions:

$$\sigma_y(z) = \sigma_1 \sqrt{\sum_{n=1}^{N} \frac{\Theta_n \phi_n^2(z)}{M_n^2 \omega_n^4} \left[1 + \frac{\pi a (1-2\xi_n)^2 (\omega_n B / 2\pi V_H)^{1.05}}{8\xi_n \left[1 + b((\omega_n B / 2\pi V_H)^2 \right]^{1.5}} \right]} \tag{35}$$

$$y(z) = \mu \sigma_y(z) \tag{36}$$

where μ is the peak factor and can be obtained according to the suggestions by Kareem and Zhou[16].

The advantage of this model is that it not only includes the contributions of higher vibration modes but also can be directly adopted by researchers or engineers because of its algebraic expression.

3. Concluding remarks

.According to the analysis above, some conclusions are summarized below:

(1) A practical higher mode generalized force spectrum (HMGFS) model is deduced on the basis of fundamental mode generalized force spectrum (FMGFS) obtained from a wind tunnel and adopting the height-independent fluctuating wind power spectral density and Shiotami-Type spatial coherence function.

(2) Based on the random vibration theory, a practical algebraic formula, in which the higher mode contributions can be included, for evaluating the RMS value of the displacement response is further derived through proper simplification.

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