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A SIMPLIFIED OPTIMIZATION STRATEGY FOR NONLINEAR TUNED MASS DAMPER IN STRUCTURAL VIBRATION CONTROL

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ABSTRACT

In this paper, a simplified optimization strategy for the nonlinear tuned mass damper (TMD) is presented, and the optimal parameter setting can be simply determined, by which the nonlinear TMD is effective over a wide frequency range. In the given numerical model, the nonlinear TMD is attached to the structure, which is represented by a single-degree-of-freedom system, and the environmental load is assumed to be the Gaussian white noise process. Governing differential equations of motion of the coupled structure-TMD system are derived, and the equivalent linearization method is introduced in the numerical calculation. The standard deviation of the structural displacement is adopted as the optimized objective function. Furthermore, it is pointed out that the response of the system can be controlled in a case of multiple probable steady-state processes caused by the nonlinearity of the stiffness element. Different from the linear TMD, the performance of the nonlinear TMD may be influenced by the excitation. Thus, the performance sensitivity of optimal nonlinear TMD is investigated with different excitation intensities and structural damping ratios. The results show that the sensitivity may limit the engineering applications of nonlinear TMD.

Key Words: Tuned mass damper, nonlinear stiffness element, equivalent linearization, optimization strategy, sensitivity

I. INTRODUCTION

In recent years, many high rise buildings, towers, chimneys, and bridges, characterized by flexibility and light damping have been constructed. These may experience large vibrations when subjected to environmental loads such as wind, wave, and earthquake excitation, and the vibration may lead to fatigue damage or structural collapse. Thus, some measures for vibration control are often taken in the design phase, or by retrofitting an existing structure, to reduce the structural responses. The common sense approach to control the structural vibration consists of adding damping, either passively or actively. The damping dissipates some of the input vibration energy of a structure by transforming it to heat or transferring it indirectly to any connected energy dissipation devices.

Structural vibration control, generally classified as active, semi-active and passive control, is an advanced technology in engineering. It enhances human comfort by reducing the excessive structural vibration with energy dissipation devices or control systems in structures. Compared with active and semi-active controls, the passive control devices have received significant attention, with the advantages of low cost, no power requirements, and easy installation. The passive control devices include base isolation, viscous damper, and tuned mass damper (TMD). Among the passive techniques, the TMD technique has been investigated and proven to be efficient, when it is properly tuned, for both wind and earthquake excitation. Furthermore, it has been installed in several structures around the world. Representative examples of this technique's engineering application are: the Centrepoint Tower in Australia (TMD, 1987), John Hancock Tower in America (TMD, 1975), Citicorp Center in America (TMD, 1980), and Chiba Tower in Japan (TMD, 1992).

The TMD is an additional mass attached to the primary structure by a spring and a damper in parallel. It was first studied by Frahm (1909) in order to reduce the vibration of a primary structure without damping. Since then, many studies have been carried out on the effectiveness of linear TMD on the vibration control response of linear structures [1–3]. In order to achieve the optimal vibration control of a structure, the stiffness and damping coefficients of the TMD should be chosen appropriately. A number of optimal solutions of the TMD's parameters for various types of excitations and various optimization objectives have been given. For the case of a primary structure without damping, the fixed point theory has been proposed by Den Hartog [1], in which the excitation was assumed to be harmonic. Warburton and Ayorinde also listed the closed-form expressions of the optimal parameters of the TMD under different types of excitation [2]. For the structure with damping characteristics, there is not usually a closed-form expression in terms of structural parameters. As the external excitation frequency is generally

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not fixed, the linear TMD poses problems that while the resonant peaks can become very steep, the response in the neighborhood of existing structural resonant frequencies may be adversely affected. At the same time, it is difficult to obtain exact dynamic solutions of the structure in practical engineering because of the inevitable perturbation of structural physical parameters, so the designed optimal control effect of linear TMD is often hard to achieve. Due to the shortcomings of linear TMD, some researchers have proposed many kinds of improved TMD for providing a better performance in vibration control, such as multiple tuned mass dampers (MTMD) and double tuned mass dampers (DTMD). Igusa has analyzed the vibration control capabilities of multiple tuned mass dampers with natural frequencies distributed over a frequency range [4,5], which is also effective in a wider frequency range than the linear TMD.

In the research mentioned above, the stiffness and damping elements connected to the additional mass are assumed to be linear. Since the 1980s, several researchers have also studied the performance of nonlinear TMD for structural vibration suppression, in which the stiffness or the damping elements, or both, were assumed to behave according to the given nonlinear law. The introduction of the nonlinear property in the TMD device would improve the performance of linear TMD to a certain extent. Roberson analyzed the performance of the nonlinear TMD with a Duffing spring, and the study shows that the nonlinear TMD is able to gain a wider vibration suppression bandwidth than the linear TMD [6]. Carter and Lin's work indicates that the TMD with a softening spring connected to the structure with a characteristic hardening spring, is more effective [7]. Hunt and Nrssen investigated the response of a structure when a nonlinear softening Belleville spring was used in the TMD device and the result shows that the vibration suppression bandwidth could be doubled by this means [8]. Inaudi and Kelly studied the nonlinear TMD in which the friction damper is adopted as a means of energy dissipation [9]. Abé proposed a design method of nonlinear TMD for structures with bilinear hysteresis subjected to harmonic excitation [10]. In Ricciardelli's work [11], the closed-form expressions of optimal parameters for the TMD with friction damper and amplitudes of multiple steady-state vibration were presented. Rüdinger has studied the optimal TMD with the nonlinear viscous power law damping under white noise excitation [12,13]. According to the literature above, it can be considered that the nonlinear TMD is more effective than the linear TMD over a wide frequency range. However, another general point concerning the nonlinear TMD is that the nonlinearity may lead to dangerous instabilities and unwanted consequences, which in some cases may result in amplification rather than reduction of the vibration amplitudes. Some researchers have also pointed out that the nonlinear system vibration can be controlled in the intended steady state through active or passive techniques, known as chaos control. Thus, the unfavorable effect due to the nonlinearity can also be avoided by appropriate parameter setting. Meanwhile, while the performance of the linear TMD has no correlation with the excitation intensity and structural damping, the actual response and optimal parameter values of the nonlinear TMD would depend on both excitation intensity and structural damping, which may limit its engineering application.

In this paper, the TMD with Duffing spring is taken into account. The governing differential equations of motion for the coupled structure-nonlinear TMD system are derived and solved under the Gaussian white noise excitation. The system is analyzed by the equivalent linearization method with the area under the square of absolute value of the frequency response function as the objective, which is proportional to the variance of the response (square of the standard deviation). The equivalent linearization method is adopted as the calculation method of the standard deviation of the response, which is accurate enough for the weakly nonlinear case. Meanwhile, a simplified optimization strategy for the nonlinear TMD is proposed, according to which the nonlinear TMD is designed effectively in a wider frequency range. In addition, the phenomenon of multiple steady-state response due to the nonlinearity is investigated based on the random vibration theory. At the end of this paper, a sensitivity study is also conducted to investigate the influence of the excitation intensity and structural damping on the performance of nonlinear TMD.

II. THEORY MODEL OF NONLINEAR TUNED MASS DAMPER

A schematic model of the nonlinear TMD is given in Fig. 1. The nonlinear TMD considered in this paper consists of a lumped mass, nonlinear stiffness element and linear viscous damping element. The equations of motion of the structure-nonlinear TMD system can be written as:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = f(t) + V(x_2, \dot{x}_2) \tag{1a}$$

$$m_2(\ddot{x}_1 + \ddot{x}_2) + V(x_2, \dot{x}_2) = g(t), \tag{1b}$$

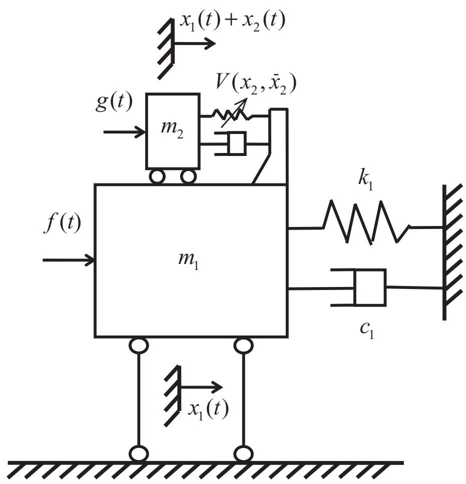


Fig. 1. Schematic model of the structure -nonlinear TMD system.

where m_1, c_1 and k_1 are the mass, damping coefficient and stiffness coefficient of the structure respectively; x_1 is the relative displacement of the structure with respect to the ground; m_2 is the mass of nonlinear TMD; x_2 is the relative displacement of nonlinear TMD with respect to the structure; $V(x_2, \dot{x}_2)$ is the force provided by the stiffness and damping element connecting structure and nonlinear TMD, which can be expressed in a type of linear or nonlinear formulation; $f(t)$ and $g(t)$ are the environmental loads applied on the structure and nonlinear TMD respectively, for wind load: $f(t) \neq 0$, $g(t) = 0$; for earthquake load in which both masses are excited: $f(t) = -m_1\ddot{x}_g$, $g(t) = -m_2\ddot{x}_g$, $\ddot{x}_g$ represents the absolute acceleration of ground motion.

The earthquake excitation is assumed to be a Gaussian white noise process given by:

$$E[\ddot{x}_g(t)\ddot{x}_g(t+\Delta t)] = 2\pi S_0 \delta(\Delta t) \tag{2}$$

in which $E[]$ is the mean value operator, $\delta()$ is the Dirac delta function, and S_0 is the intensity of the Gaussian white noise process $\ddot{x}_g(t)$, that means $S_{\ddot{x}_g} = S_0$.

Initially, it is supposed that there is no TMD on the structure, and the equation of motion can be established as follows:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = -m_1\ddot{x}_g \tag{3}$$

Considering the single degree of freedom system described in (3), the transfer function can be easily identified. As the power spectral density function (PSDF) of external excitation $\ddot{x}_g(t)$ is $S_{\ddot{x}_g} = S_0$, the PSDF of structural response can be further calculated. By the integration of PSDF, the standard deviation of structural response can be derived and written in the following expression:

$$\sigma_{x_1} = x_0 = \sqrt{\frac{\pi S_{\ddot{x}_g}}{2\zeta_1\omega_1^3}}, \tag{4}$$

in which $\omega_1 = \sqrt{k_1/m_1}$ is the frequency of structure, $\zeta_1 = c_1/2m_1\omega_1$ is the damping ratio of structure, $S_{\ddot{x}_g} = S_0$. Choosing ω_1^{-1} as the time scale, x_0 as the response scale, the non-dimensional displacements and time are given by:

$$y_i = \frac{x_i}{x_0}, \quad \tau = \omega_1 t. \tag{5}$$

Then $\dot{x}_i(t)$ and $\ddot{x}_i(t)$ can be respectively expressed as:

$$\dot{x}_i(t) = y_i(\tau)' x_0\omega_1, \quad \ddot{x}_i(t) = y_i(\tau)'' x_0\omega_1^2, \tag{6}$$

in which the overdots denote the derivative with respect to time t while the primes represent the derivative with respect to non-dimensional time τ. The resulting non-dimensional equations that govern the motion of the structure-nonlinear TMD system are given by:

$$(1+\mu)y_1'' + \mu y_2'' + 2\zeta_1 y_1' + y_1 = f_n(\tau) + g_n(\tau) \tag{7a}$$

$$\mu y_1'' + \mu y_2'' + V_n(y_2, y_2') = g_n(\tau), \tag{7b}$$

in which $\mu = m_2/m_1$ is the mass ratio between the nonlinear TMD and primary structure; ω_1 and ζ_1 are the frequency and damping ratio of the structure respectively; the non-dimensional forces $f_n(\tau)$, $g_n(\tau)$ and $V_n(y_2, y_2')$ are given by:

$$f_n(\tau) = f(t)/m_1\omega_1^2 x_0, \quad g_n(\tau) = g(t)/m_1\omega_1^2 x_0, \\ V_n(y_2, y_2') = V(x_2, \dot{x}_2)/m_1\omega_1^2 x_0, \tag{8a-c}$$

in which $f(t) = -m_1\ddot{x}_g$, $g(t) = -m_2\ddot{x}_g$, and $S_{\ddot{x}_g} = S_0$. Thus, by dimensional analysis the intensity of non-dimensional Gaussian white noise excitation $f_n(\tau)$ and $g_n(\tau)$ can be expressed as:

$$S_{f_n(\tau)} = \frac{2\zeta_1}{\pi}, \quad S_{g_n(\tau)} = \mu^2 \frac{2\zeta_1}{\pi}. \tag{9a, b}$$

The given non-dimensional equations above describe the dynamic characteristics of the coupled structure- nonlinear TMD system, and it can be seen that the non-dimensional displacement of structure accounts for the vibration suppression effect of the nonlinear TMD. It is also known that the standard deviation of structural non-dimensional displacement is unity in this formulation, when the TMD is absent. Here, the nonlinear stiffness element connected to the primary structure and secondary TMD mass is assumed to be the Duffing spring, and the damping element is the linear viscous damper. Thus, $V(x_2, \dot{x}_2)$ in (1a,b) is described as follows:

$$V(x_2, \dot{x}_2) = k_2x_2 + \tilde{k}_2x_2^3 + c_2\dot{x}_2 \tag{10}$$

where the Duffing spring is characterized by the stiffness coefficients k_2 and $\tilde{k}_2$. According to the non-dimensional (8c), $V(x_2, \dot{x}_2)$ can be replaced by $V_n(y_2, y_2')$ as follows:

$$V_n(y_2, y_2') = V(x_2, \dot{x}_2)/k_1x_0 = \frac{k_2}{k_1}y_2 + \frac{\tilde{k}_2}{k_1}x_0^2y_2^3 + \frac{c_2\omega_1}{k_1}y_2'. \tag{11}$$

III. EQUIVALENT LINEARIZATION METHOD

In the nonlinear TMD, the analytical solution of a structural response is usually difficult to obtain. Thus, some approximate methods have been proposed, a representative of which is the equivalent linearization method characterised by its simplicity in computation. In the equivalent linearization method, a set of equivalent linear equations are established to replace the nonlinear equations of motion.

The difference between the nonlinear element representing the Duffing spring and the equivalent linear element is defined as Δ, which can be given by

$$\Delta = \frac{\tilde{k}_2}{k_1} x_0^2 y_2^3 - \tilde{k}_{2,equ} y_2. \tag{12}$$

The relationship between the nonlinear stiffness coefficient $\tilde{k}_2$ and equivalent linear stiffness coefficient $\tilde{k}_{2,equ}$ is established by minimizing the mean square value of the difference Δ according to the least square method [14]. Here the excitation and response are both assumed to be Gaussian processes, so the response process represented by the non-dimensional response y_2 can be confirmed by two parameters: mean value and mean square value of y_2. The expression of the mean square value of the difference Δ can, therefore, be described as:

$$E[\Delta^2] = \int_{-\infty}^{\infty} \left[\frac{\tilde{k}_2}{k} x_0^2 y_2^3 - \tilde{k}_{2,equ} y_2 \right]^2 p(y_2) dy_2,$$
$$p(y_2) = \frac{1}{\sqrt{2\pi}\sigma_{y_2}} \exp\left(\frac{y_2^2}{2\sigma_{y_2}^2} \right). \tag{13a,b}$$

According to (13a,b), the equivalent linear stiffness coefficient $\tilde{k}_{2,equ}$ can be given by

$$\tilde{k}_{2,equ} = 3 \frac{\tilde{k}_2}{k_1} x_0^2 (a_{y_2} + \sigma_{y_2}^2), \tag{14}$$

in which a_{y_2} denotes the mean response value of the TMD, which is equal to zero for the equivalent linear system under Gaussian stationary excitation. $\sigma_{y_2}^2$ is the mean square value of non-dimensional response y_2.

Introducing the equivalent linear coefficient of (14) into (10), the equations of motion in (7a,b) can be rewritten as

$$(1+\mu) y_1'' + \mu y_2'' + 2\zeta_1 y_1' + y_1 = f_n(\tau) + g_n(\tau) \tag{15a}$$

$$\mu y_1'' + \mu y_2'' + V_{n,equ}(y_2, y_2') = g_n(\tau), \tag{15b}$$

where $V_{n,equ}(y_2, y_2')$ is the equivalent linear force provided by the equivalent linear spring and damper, and can be described by

$$V_{n,equ}(y_2, y_2') = k_{2,equ} y_2 + c_{2,equ} y_2', \quad k_{2,equ} = \frac{k_2}{k_1} + \tilde{k}_{2,equ},$$
$$c_{2,equ} = \frac{c_2 \omega_1}{k_1}, \tag{16a,b}$$

in which $\tilde{k}_{2,equ} = 3\tilde{k}_2 x_0^2 \sigma_{y_2}^2 / k_1$ is given in (14). In (15a,b), all the quantities are non-dimensional and y_1 can describe the vibration suppression effect of the TMD, so non-dimensional equations (15a,b) are more advantageous than (1a,b).

Assume the harmonic excitation and response expressions as follows:

$$y_i = Y_i e^{i\omega\tau}, \quad y_i' = i\omega Y_i e^{i\omega\tau}, \quad y_i'' = -\omega^2 Y_i e^{i\omega\tau},$$
$$f_n(\tau) = f_{n,0} e^{i\omega\tau}, \quad g_n(\tau) = g_{n,0} e^{i\omega\tau} = \mu f_{n,0} e^{i\omega\tau}, \tag{17a-e}$$

where i is the imaginary unit; ω means the non-dimensional excitation frequency; $f_{n,0}$ and $g_{n,0}$ are the amplitudes of the non-dimensional excitation $f_n(\tau)$ and $g_n(\tau)$, respectively. Substituting (17a-e) into (15a,b), the frequency response functions are determined, thus:

$$H_{y_1}(\omega) = \frac{-\mu\omega^2 + (1+\mu) c_{2,equ} i\omega + (1+\mu) k_{2,equ}}{[-(1+\mu)\omega^2 + 2\zeta_1 i\omega + 1](-\mu\omega^2 + c_{2,equ} i\omega + k_{2,equ}) - \mu^2\omega^4} \tag{18a}$$

$$H_{y_2}(\omega) = \frac{\mu(2\zeta_1 i\omega + 1)}{[-(1+\mu)\omega^2 + 2\zeta_1 i\omega + 1](-\mu\omega^2 + c_{2,equ} i\omega + k_{2,equ}) - \mu^2\omega^4}, \tag{18b}$$

in which $H_{y1}(\omega)$ represents the ratio between output and input of the structure, and $H_{y2}(\omega)$ represents the ratio between output and input of TMD. Here non-dimensional parameters $c_{2,equ}$ and $k_{2,equ}$ are adopted in the expression of (18a,b), and can facilitate following derivation. The mean square values of the stationary response can be evaluated as

$$\sigma_{y_1}^2 = S_F \int_{-\infty}^{\infty} |H_{y_1}(\omega)|^2 d\omega, \quad \sigma_{y_2}^2 = S_F \int_{-\infty}^{\infty} |H_{y_2}(\omega)|^2 d\omega, \tag{19a,b}$$

in which $S_F = S_{fn}$ is the power spectrum density function of external loads. From (19a,b) it can be seen that while the excitation is a Gaussian white noise process, the excitation PSDF S_F is a constant value and the frequency response functions can, to a certain extent, describe the relative magnitude of mean square values of structural responses, so only the frequency response functions are needed here.

In the present case, the integral may be evaluated analytically. The closed form expression first developed by Crandall and Mark [15], was later used as a standard procedure by Warburton [16,17], Rüdinger [12,13], Krenk and Hogsberg [18,19]. Also the procedure is closely related to the work by Rüdinger in [12] and [13]. Using the formulas given, the integral can be obtained by:

$$I = \int_{-\infty}^{\infty} |H_{y_i}(\omega)|^2 d\omega = \frac{M}{N} \tag{20a}$$

$$M = \pi\{A_0 B_3^2 (A_0 A_3 - A_1 A_2) + A_0 A_1 A_4 (2B_1 B_3 - B_2^2) - A_0 A_3 A_4 (B_1^2 - 2B_0 B_2) + A_4 B_0^2 (A_1 A_4 - A_2 A_3)\} \tag{20b}$$

$$N = A_0 A_4 (A_0 A_3^2 + A_1^2 A_4 - A_1 A_2 A_3) \tag{20c}$$

in which

$$A_0 = k_{2,equ}, \quad A_1 = 2\zeta_1 k_{2,equ} + c_{2,equ},$$
$$A_2 = (1+\mu) k_{2,equ} + 2\zeta_1 c_{2,equ} + \mu,$$
$$A_3 = (1+\mu) c_{2,equ} + 2\zeta_1 \mu, \quad A_4 = \mu \tag{21a-e}$$

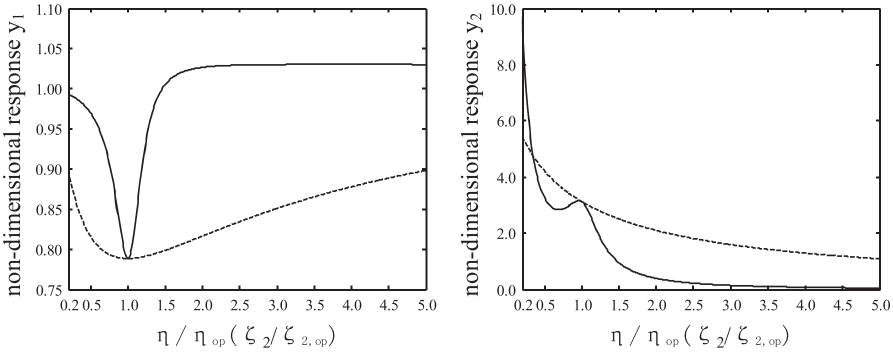


Fig. 2. Effect of perturbation of frequency and damping ratio on non-dimensional response of (a) structure and (b) linear TMD, $m_1 = 1$, $\zeta_1 = 0.05$, $\mu = 0.03$, frequency ratio perturbation —, damping ratio perturbation —.

for y_1

$$B_0 = (1+\mu)k_{2,equ}, \quad B_1 = (1+\mu)c_{2,equ}, \quad B_2 = \mu, \quad B_3 = 0 \tag{22a-d}$$

for y_2

$$B_0 = \mu, \quad B_1 = 2\zeta_1\mu, \quad B_2 = 0, \quad B_3 = 0. \tag{23a-d}$$

As seen in (14), the equivalent linear stiffness coefficient $\tilde{k}_{2,equ}$ depends on the response $\sigma^2_{\tilde{y}_2}$ of the nonlinear TMD. The stationary responses $\sigma^2_{y_1}$ and $\sigma^2_{\tilde{y}_2}$ are, however, obtained by (19–23) while given the equivalent linear coefficient $k_{2,equ}$. Therefore, the solution is an iterative process, which can be described as: (i) determine a initial value of the equivalent linear coefficient; (ii) calculate the mean square response of the equivalent linear system by (19–23); (iii) obtain a new equivalent linear coefficient according to (14); and (iv) compare the initial and new values of the equivalent linear coefficient of the stiffness element, and repeat the above steps until the results converge.

IV. OPTIMIZATION STRATEGY IN FREQUENCY DOMAIN

4.1 Linear tuned mass damper

The non-dimensional equations of motion of structure-linear TMD system can be similarly established as:

$$(1+\mu)y_1'' + \mu y_2'' + 2\zeta_1 y_1' + y_1 = f_n(\tau) + g_n(\tau) \tag{24a}$$

$$y_1'' + y_2'' + \eta^2 y_2 + 2\zeta_2\eta y_2' = f_n(\tau), \tag{24b}$$

in which $\eta = \omega_2/\omega_1$ is frequency ratio between TMD and structure, ω_1 is circular the frequency of structure and ω_2 is the circulaer frequency of TMD; ζ_2 is damping ratio of damping element of the TMD; other symbols have been defined in (7a,b). Compared with the non-dimensional equations of motion of structure-nonlinear TMD system in (15a,b), it can be obtained similarly

$$k_{2,equ} = \mu\eta^2, \quad c_{2,equ} = 2\mu\zeta_2\eta. \tag{25a,b}$$

Here, the mean square response $\sigma^2_{y_1}$ can be obtained according to (19–23), in which the integral can be evaluated analytically using formulae proposed by Gradshteyn and Ryzhik [20]. The optimal values of ζ_2 and η can be determined by minimizing the mean square response of the structure. This corresponds to the solution of the two following equations:

$$\frac{\partial\sigma^2_{y_1}}{\partial\eta} = 0, \quad \frac{\partial\sigma^2_{y_1}}{\partial\zeta_2} = 0 \tag{26a,b}$$

If the structural damping is neglected, the close-form solution of (26a,b) can be obtained. When the environment load is the ground motion, the closed-form solution for a no damping situation is written as follows, which has been proposed in reference [2]:

$$\eta_{op} = \frac{(1-\mu/2)^{\frac{1}{2}}}{1+\mu}, \quad \zeta_{2,op} = \sqrt{\frac{\mu(1-\mu/4)}{4(1+\mu)(1-\mu/2)}}. \tag{27a,b}$$

If the structural damping isn't equal to zero, the closed-form expressions in (27a,b) are generally not applicable. The optimal values of parameters of nonlinear TMD can be obtained by numerical methods. However, some researchers have investigated the influence of structural damping on the optimal solution, and the results show that while structural damping is light enough the influence can be seen to be negligible, so the expressions of η_{op} and $\zeta_{2,op}$ in (27a,b) can also be used as an approximation of the exact solution for the light damping structure.

Suggested by the previous work, it is known that the robustness of linear TMD is poor, and a small perturbation of optimal frequency ratio usually leads to a sharp decrease of the performance of linear TMD. In this paper, the parameters of structure-linear TMD system are assumed to be: the mass $m_1 = 1$, damping ratio $\zeta_1 = 0.05$, mass ratio $\mu = 0.03$, and as has been seen in (24a,b), the non-dimensional response has no correlation with the absolute frequency of the primary structure under Gaussian white noise excitation. Due to the unneglectable damping of the structure, numerical methods can be used for the optimal frequency and damping ratio of linear TMD. Fig. 2 describes the trend of the structure-linear TMD system's response with respect to perturbation of the frequency ratio and damping ratio. As shown in Fig. 2a, the non-dimensional structural response y_1, which represents the structural vibration reduction ratio, varies sharply around the optimal value with frequency ratio perturbation and gently with damping ratio perturbation. In Fig. 2b, non-dimensional response y_2 of TMD decreases obviously with increasing damping ratio, and varies gently with small perturbation of the optimal frequency ratio, especially in the frequency range that is a little lower than the optimal value.

4.2 Nonlinear tuned mass damper

4.2.1 Conventional optimization strategy

Different from the linear TMD, the performance of the nonlinear TMD can be influenced by its own seismic response, so there are different equivalent linear TMD systems corresponding to different response states of secondary mass, while the nonlinear TMD has the same parameter setting, which can be seen from (14). Researchers have studied many kinds of nonlinear TMD, in which the nonlinear element may be a stiffness element, a damping element, or both. Several valuable results have been given, one of which is that the introduction of nonlinearity can distinctly improve the performance of TMD and broaden the effective frequency range for structural vibration suppression. Thereby, the nonlinear TMD is generally more applicable than the linear TMD, despite its sensitivity to the excitation and structural physical parameters. In previous studies, the environmental excitation is usually assumed to be a deterministic excitation, such as sine wave etc, and the seismic response of structure-nonlinear TMD system can be determined in a deterministic category. Here, what is different is that the external load is assumed to be a random excitation, so the structure-nonlinear TMD system should be analyzed based on the random theory. Similarly, the response of the structure under random excitation can also be controlled over a wider frequency range by an appropriate parameter setting of the nonlinear TMD.

Generally, the optimization strategy of the nonlinear TMD is similar to the linear TMD, however, while the nonlinear TMD is effective in a wider frequency range, the non-dimensional response for the structure should be controlled in the frequency range. So it is a multiple objective optimization problem. The min max optimization strategy is applied in the parameter optimization of nonlinear TMD, which can be described as

$$f_{obj} = \min\max\left\{\sigma^2_{y_1,\omega_1}, \omega_1 \in [\omega_l, \omega_u]\right\}, \tag{28}$$

in which ω_1 is the structural frequency varying in a frequency range, ω_l is the lower bound of the frequency range, ω_u is the upper bound of the frequency range, and $\sigma^2_{y_1,\omega_1}$ corresponds to the mean square response of the structure when the structural frequency varies in the range $\omega_1 \in [\omega_l, \omega_u]$. Here the upper and lower frequency boundaries are set just to point out the perturbation range of structural frequency. The optimal parameters of nonlinear TMD can be obtained by minimizing the objective function f_{obj} in (28) by numerical method, and according to the obtained optimal parameter the minimum value of the maximum non-dimensional structural response in the frequency range can be obtained. However, the min max optimization strategy for the nonlinear TMD is cumbersome and time consuming, which may restrict its engineering application.

4.2.2 Simplified optimization strategy

For weakly nonlinear systems, the equivalent linearization method is usually adopted as an approximate approach. According to this method, the nonlinear TMD can be equivalently substituted by a series of linear TMDs and, based on the properties of the linear TMD referred to in the previous section, a new optimization strategy for nonlinear TMD is presented here, which is more simple and applicable.

As shown in Section III, it is known that the equivalent linear TMD differs along with the changing response of secondary mass. Based on this point a simplified optimization strategy for the nonlinear TMD effective in a wider frequency range can be proposed. Designed by the optimization strategy, the frequency of nonlinear TMD changes along with the perturbation of structural frequency so that the frequency ratio between the structure and the TMD changes little around the optimal value. Therefore, the perturbation of structural frequency has little influence on the performance of nonlinear TMD, and the vibration reduction in a wider frequency range can be achieved. It should also be mentioned that the nonlinear TMD consisting of some types of nonlinear stiffness elements is effective for the vibration control over a wide frequency range, while some other types of nonlinear stiffness elements are not suitable.

The model of nonlinear TMD consists of the small mass, nonlinear Duffing spring and linear viscous damper. According to the idea of optimization design above, by an appropriate setting of nonlinear TMD the frequency ratio of the structure-equivalent linear TMD system can deviate little from the optimal value while the structural frequency varies. However, the damping ratio will deviate from the optimal value obviously in this case, which is unfavorable. It can be seen from the previous section that a proper deviation of the damping ratio from the optimal value has less influence on the effect of vibration reduction compared with the frequency ratio deviation. From another more favorable viewpoint, the appropriate setting of the nonlinear TMD should cause deviation of the damping ratio from the optimal value which is insensitive, instead of the deviation of frequency ratio which is sensitive. The "automatic adjustment" characteristic of nonlinear TMD along with the structural frequency shifting is similar to the active and semi-active control devices, what is different is that the nonlinear TMD is more applicable and has no external power requirement. However, obviously the corresponding adjustable degree of the nonlinear TMD is more limited compared with active and semi-active control, but more applicable compared with passive linear TMD, and by the proposed new optimization strategy in this paper the nonlinear TMD can work in a wider frequency range. Yet, there are still some other disadvantages for nonlinear TMD introduced by the nonlinear element, such as its sensitivity to excitation and structural damping characteristic, the stability problem and multiple steady-state response. Despite these problems, the nonlinear TMD is more applicable than linear TMD, and is a lower cost alternative to the active devices. The perturbation of structural frequency leads to the variation of damping ratio of equivalent linear TMD and, if the deviation of damping ratio from the optimal value is too large, the performance of the nonlinear TMD will deteriorate obviously. Conversely, the magnitude of damping ratio deviation from the optimal value will limit the maximum perturbation range of structural frequency, and within the range the nonlinear TMD is effective. While structural frequency perturbation exists in a wide range, the ideal condition for the nonlinear TMD effective in the range is that the frequency ratio and damping ratio equal the optimal values simultaneously, which is usually impossible. In view of that the performance of nonlinear TMD is sensitive to the frequency ratio deviation from the optimal value and less sensitive to the damping ratio deviation, an appropriate setting of nonlinear TMD is that the frequency ratio approximately equals the optimal value and the damping ratio shifts in a proper range while the structural frequency varies. Accordingly a new optimization strategy is proposed, which can be described as

$$\frac{k_2}{m_1\omega_1^2}+3\frac{\tilde{k}_2}{m_1\omega_1^2}x_0^2\sigma_{y_2}^2=\mu\eta_{op}^2,\quad \frac{c_2}{m_1\omega_1}=2\mu\eta_{op}\zeta_2,\quad \omega_1\in[\omega_l,\omega_u],\quad \zeta_2\in[\zeta_l,\zeta_u], \tag{29a,b}$$

in which the structural frequency varies in the range $[\omega_1,\omega_u]$, the damping ratio of equivalent linear TMD varies in the range $[\zeta_1,\zeta_u]$, η_{op} is the optimal value of frequency ratio obtained by numerical method, and $\sigma_{y_2}^2$ represents the mean square response of secondary mass. When the damping of structure is neglected, the expressions above can be rewritten as

$$\frac{k_2}{m_1\omega_1^2}+3\frac{\tilde{k}_2}{m_1\omega_1^2}x_0^2\sigma_{y_2}^2=\mu\left[\frac{(1-\mu/2)^{\frac{1}{2}}}{1+\mu}\right]^2,\quad \frac{c_2}{m_1\omega_1}=2\mu\frac{(1-\mu/2)^{\frac{1}{2}}}{1+\mu}\zeta_2 \tag{30a,b}$$

which can be regarded as an approximation of light damping structure.

Utilizing the strategy for the optimization of nonlinear TMD, first the value of c_2 can be obtained by (29a,b) making the frequency and damping ratio equal to the optimal value for the main control frequency:

$$c_2=2m_1\omega_{1,r}\mu\eta_{op}\zeta_{2,op}=2m_2\omega_{1,r}\eta_{op}\zeta_{2,op}, \tag{31}$$

in which $\omega_{1,r}$ represents the main controlled frequency of structure, η_{op} is the optimal frequency ratio, and $\zeta_{2,op}$ is the optimal damping ratio for the linear TMD. Assuming the frequency ratio of equivalent linear TMD is approximately equal to the optimal value while the structural frequency varies, the equivalent linear damping ratio varies as follows:

$$\zeta_{2,i}=\frac{c_2}{2m_2\omega_{1,i}\eta_{op}},\quad \omega_{1,i}\in[\omega_l,\omega_u], \tag{32}$$

in which $\omega_{1,i}$ denotes the structural frequency in the range $[\omega_1,\omega_u]$ and $\zeta_{2,i}$ is the corresponding changing damping ratio. As the damping coefficient c_2 has been given by (31), the simplified optimization strategy is to confirm the stiffness coefficients k_2 and k_2' , so a series of equivalent linear TMD systems can be established with the frequency ratio equal to the optimal value and damping ratio equal to the value calculated by (32). The equivalent linear TMD series can be written as:

$$I_{equ}=\{(\zeta_{2,1},\eta_{op}),\ldots,(\zeta_{2,i},\eta_{op}),\ldots,(\zeta_{2,n},\eta_{op})\}. \tag{33}$$

In order to ensure that the frequency ratio approximately equals the optimal value, (29a) should be satisfied in the perturbation range of structural frequency, which generally means to get the solution of n equations with two arguments. The least square method can be used here, by which the solution with minimum of square sum of error is calculated. A matrix equation can be given by

$$A_kP_k=B_k, \tag{34}$$

in which $A_k=[1/m_1\omega_{1,1}^2, 3(x_0^2)^{(1)}\sigma_{y2,1}^2/m_1\omega_1^2;\ldots,\ldots;1/m_1\omega_{1,n}^2, 3(x_0^2)^{(n)}\sigma_{y2.n}^2/m_1\omega_{1,n}^2]$ is the coefficient matrix, $P_k=[k_2,\tilde{k}_2]^T$ is the nonlinear stiffness coefficient vector of TMD, and B_k is the constant vector with the elements equal to $\mu\eta_{op}^2$. It's obvious that the number of arguments are less than that of constraint equations so that a trial based on the least square method is made to fit (29a). Defining $e_k=A_kP_k-B_k$, and the transposed matrix $e_k^T=P_k^TA_k^T-B_k^T$, one has

$$\begin{aligned}S_k&=e_k^Te_k=P_k^TA_k^TA_kP_k-B_k^TA_kP_k^T-P_k^TA_k^TB_k+B_k^TB_k\\&=P_k^TH_kP_k-2P_K^TG_k^T+B_k^TB_k\end{aligned} \tag{35a}$$

$$\frac{\partial S_k}{\partial P_{k(n)}}=2[0\quad 0\quad\cdots\quad 1\quad\cdots\quad 0][H_kP_k-G_k]=0,\quad n=1,2 \tag{35b}$$

$$H_kP_k=G_k,\quad H_k=A_k^TA_k,\quad G_k=A_k^TB_k, \tag{35c-e}$$

in which the optimal values of nonlinear TMD's parameters are given in a close-form expression. It is necessary to substitute the solution of nonlinear TMD obtained by (35a-e) into (29a) to check if the left-hand side equals the right-hand side, or not. If the difference between the left-hand and right-hand sides of equation is small, it is considered that the setting of nonlinear TMD satisfies the objective of vibration suppression of structure in the given frequency range; if the difference is large, the design strategy is not suitable in the frequency range, while it is likely suitable in a relatively narrow frequency range. The frequency ratio calculated according to (29a) is written as:

$$\eta_i = \sqrt{\frac{k_2}{\mu m_1 \omega_{1,i}^2} + 3\frac{\tilde{k}_2}{\mu m_1 \omega_{1,i}^2} x_0^2 \sigma_{y2,i}^2}\,, \tag{36}$$

in which $\sigma_{y2,i}^2$ corresponds to the equivalent linear TMD series I_{equ}, because the non-dimensional response of secondary mass is insensitive to frequency ratio as have been shown in Fig. 2b. The evaluation index for the difference between the left-hand and right-hand sides of equation can be selected as follows:

$$e_\eta = \max\left\{abs\left(\frac{\eta_i - \eta_{op}}{\eta_{op}}\right), i = 1, \ldots, n\right\}. \tag{37}$$

Generally, the new simplified optimization strategy for nonlinear TMD can be described as follows:

1. Choose a perturbation range of frequency, calculate the damping efficient c_2 by (31), and establish a series of equivalent linear TMD systems, in which the frequency ratio approximately equals the optimal value, and the damping ratio is calculated by (32).
2. Solve (29a) by the least square method, and obtain the parameters of nonlinear TMD by (35a-e).
3. Analyze the difference by (37) to check if the left-hand side is approximately equal to the right-hand side. If the difference between the left-hand and right-hand sides of equation is small, the setting of nonlinear TMD satisfies the objective of vibration reduction in a wide frequency range; if the difference is great, the optimization strategy is not suitable in the frequency range, and it is likely to be suitable in a relatively narrow frequency range.
4. Reduce the max perturbation range of structural frequency, and repeat steps 2–4 until the difference between left-hand and right-hand sides of (29a) is small enough.
5. Obtain the optimal value of stiffness and damping coefficients, and check the validity of the nonlinear TMD.

4.2.3 Error of structural frequency estimation

It is well known that the optimized linear TMD based on a structural model with exactly defined parameters may not be so effective, and may even fail, due to the inevitable errors in the construction, especially the perturbation of structural frequency. In this section, the new strategy is used for the design of nonlinear TMD. After assuming the main control frequency, the stiffness and damping coefficients of the nonlinear TMD is simply calculated by the new optimization strategy, and the effective frequency range is broadened compared with the linear TMD. The main controlled frequency is assumed to be $\omega_{1,r} = 20$ rad/s. Define the perturbation range of structural frequency as

$$p_{\omega_1} = abs\left(\frac{\omega_1 - \omega_{1,r}}{\omega_{1,r}}\right). \tag{37}$$

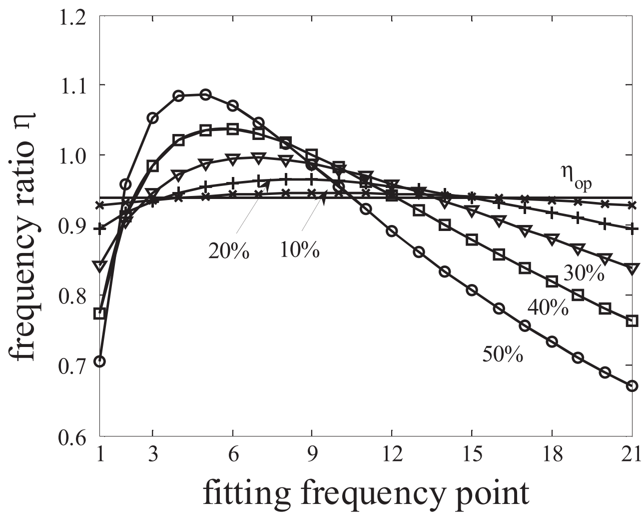


Fig. 3. Fitting frequency ratio corresponding to different frequency perturbation range.

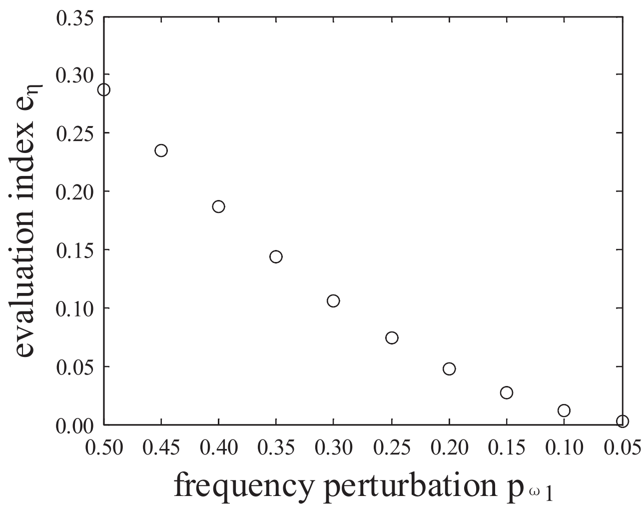


Fig. 4. Evaluation index corresponding to different frequency perturbation range.

Fig. 3 describes the difference between the optimal frequency ratio and frequency ratio calculated by (29a) for 10%, 20%, 30%, 40%, 50% perturbation range of structural frequency, respectively. Assuming that the number of fitting frequency points distributed uniformly in the perturbation range are 21, Fig. 4 shows the magnitude of evaluation index e_η for different frequency perturbation range. Accordingly it can be shown that while the perturbation frequency range is less than 10% the evaluation index e_η is small enough, meaning the left-hand side of (29a) is approximately equal to the right-hand side. The damping and stiffness coefficients of nonlinear TMD is simply obtained by the new optimization strategy: $c_2 = 0.097$, $k_2 = 20.93$, $k_2' = -8866.86$. In the non-dimensional (7) and (11), the nonlinear stiffness coefficient is $\tilde{k}_2 x_0^2 / k_1$ and linear stiffness coefficient is k_2/k_1, in which the

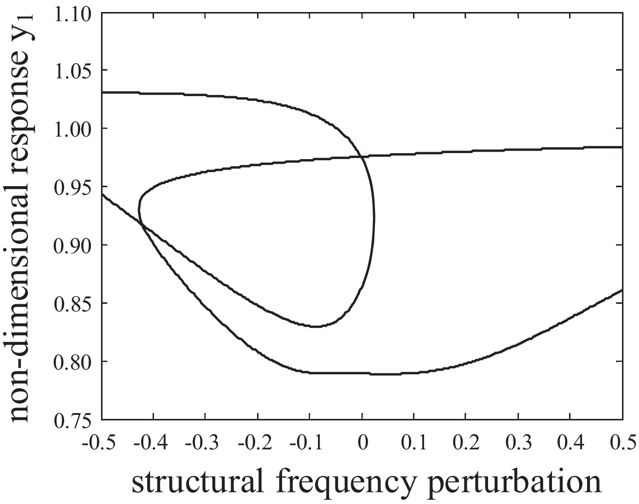


Fig. 5. Multiple steady state response of structure attached with nonlinnear TMD.

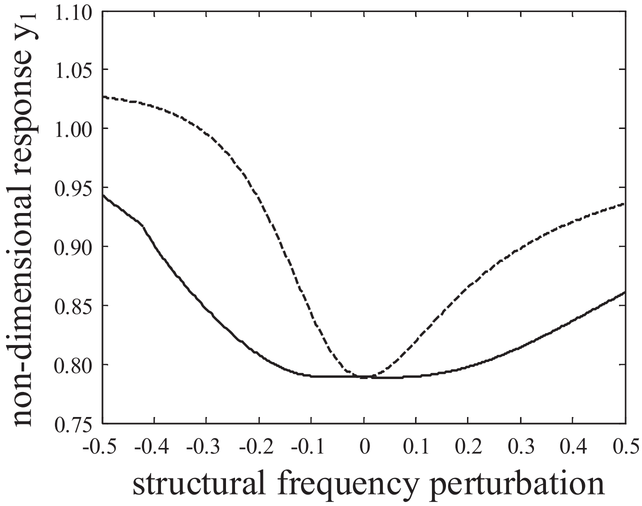


Fig. 6. Contrast of non-dimensional response of structure attached with linear TMD (—) and nonlinear TMD (---).

magnitude of $x_0 = \sqrt{\pi S_{\ddot{x}_g} / 2\zeta_1\omega_1^3}$ is usually very small. Here, x_0 is less than 0.01 in the perturbation range of structural frequency. Thus, the ratio between nonlinear stiffness coefficient and linear stiffness coefficient $\tilde{k}_2 x_0^2 / k_2$ is much smaller than 1. Therefore, the nonlinear TMD is weakly nonlinear, and the equivalent linearization method has enough accuracy.

V. STABILITY ANALYSIS

Due to the introduction of nonlinearity in the TMD system, there are usually multiple steady state responses corresponding to a parameter combination setting of nonlinear TMD. The secondary mass can suppress the structural response in some steady states, while others may have no or adverse function for structural response suppression. As shown in the previous sections, the optimized nonlinear TMD is established according to the new proposed optimization strategy in this paper and the nonlinear stiffness element is assumed to be a soft Duffing spring.

Fig. 5 gives the multiple steady state responses of a structure with attached nonlinear TMD. The nonlinear TMD can suppress the structural response in the low steady state response as shown in the figure. In order to avoid the vibration of the nonlinear TMD in an unfavorable steady state, the control techniques can be adopted to induce an intended steady state, which is known as the chaos control. Compared with the general active control techniques, the technique only needs a little transition energy for the nonlinear TMD into the favorable steady state. The research here doesn't emphasize the control technique making nonlinear TMD vibrate in the favorable steady state, so it is not introduced in this section.

In Fig. 6, the dashed line describes the influence of structural frequency perturbation on the non-dimensional response of the structure with attached linear TMD, which illustrates the performance of the linear TMD, and the solid line corresponds to the favorable steady state while the structural frequency varies in a range. It can be shown that the nonlinear TMD in the favorable steady state is more effective in a wider frequency range than the linear TMD.

VI. SENSITIVITY STUDY

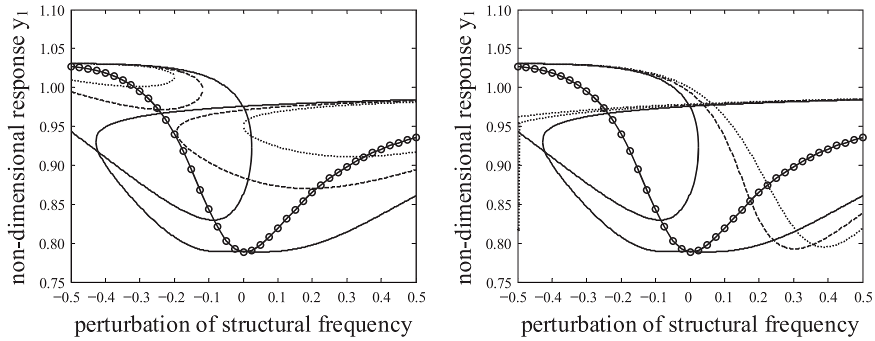
Different from the linear TMD, the performance of nonlinear TMD can be influenced by the external excitation intensity and damping ratio of structure. In this section, the sensitivity of nonlinear TMD designed by the new optimization strategy is investigated and it is pointed out that the sensitivity may limit the engineering applications of nonlinear TMD.

6.1 Influence of excitation intensity

Fig. 7 describes the influence of external excitation intensity on the non-dimensional response of the primary structure, which represents the performance of the nonlinear TMD. For the optimized nonlinear TMD designed by the strategy mentioned in the previous section, its performance would deteriorate for both the increasing and decreasing excitation intensity series. However, the influenced law is somewhat different as it can be seen in the figure. The designed nonlinear TMD is more suitable for the intensity determined excitation, such as wind excitation, the intensity of which can be approximately determined in a certain geographic region.

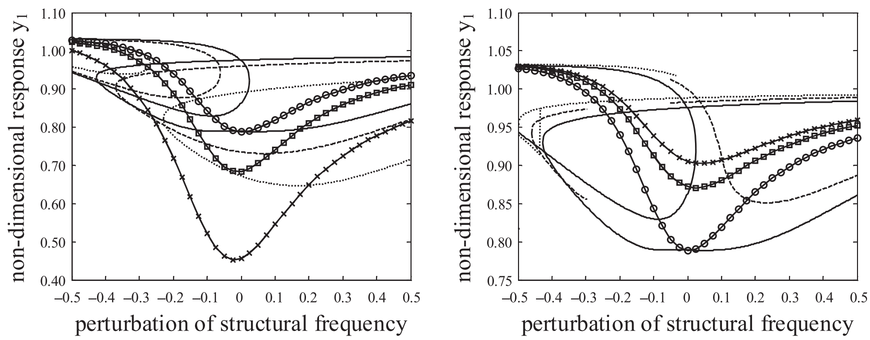
6.2 Influence of damping ratio of structure

The influence of perturbation of structural damping ratio on the performance of nonlinear TMD is investigated here. As shown in Fig. 8, the perturbation can affect



(a). S_0=0.01,—, S_0=0.015,---, S_0=0.02, ····. (b). S_0=0.01,—, S_0=0.005,---, S_0=0.0001, ····.

Fig. 7. Sensitivity of nonlinear TMD's performance for vibration suppression of structure to (a) increase and (b) decreased excitation intensity, $m_1 = 1$, $\mu = 0.03$, $\zeta_1 = 0.05$, linear TMD: —O—.



(a). nonlinear TMD: ζ_1=0.05,—, ζ_1=0.03,---, ζ_1=0.01, ····; linear TMD: ζ_1=0.05, -o- , ζ_1=0.03, -□- , ζ_1=0.01, -x- .

(b). nonlinear TMD: ζ_1=0.05,—, ζ_1=0.08,---, ζ_1=0.1, ····; linear TMD: ζ_1=0.05, -o- , ζ_1=0.08, -□- , ζ_1=0.1, -x- .

Fig. 8. Sensitivity of linear and nonlinear TMD's performance for vibration suppression of structure to (a) decreased and (b) increased structural damping ratio, $m_1 = 1$, $\mu = 0.03$.

the non-dimensional response of the structure. Compared with the linear TMD, the nonlinear TMD can broaden the effective frequency range for structural vibration suppression. However, the vibration suppression effect is more effective while the damping ratio varies, and the center frequency in the range for vibration reduction is not the main control frequency anymore. While the structural damping ratio decreases, the performance of the nonlinear TMD represented by the non-dimensional response of structure is better than the initial case, and a little worse compared with the optimized linear TMD, whereas the nonlinear TMD is effective in a wider frequency range than the linear TMD. When the structural damping ratio increases, the performance of nonlinear TMD deteriorates accordingly and even falls into failure for the large augmentation of structural response. Thereby, the perturbation of the structural damping ratio should be considered and control in a reasonable range included in the design of nonlinear TMD. For the nonlinear TMD designed by the optimization strategy described in the previous section, a larger damping ratio is favorable in the modeling of practical structures.

VII. CONCLUSIONS

In this paper, the seismic response of coupled structure-nonlinear TMD system subjected to Gaussian white noise excitation is calculated, in which the nonlinear element is assumed to be the Duffing spring. In the numerical analysis, the equivalent linearization method, which is accurate enough for the weakly nonlinear case, is used and the standard deviation of the response is adopted as the optimization index. Meanwhile, a simplified optimization strategy for the nonlinear TMD is presented, according to which the nonlinear TMD can be designed effectively in a wider frequency range than that of the linear TMD. Furthermore, the multiple steady states due to the introduction of nonlinearity are investigated, and the sensitivity of nonlinear TMD to the excitation intensity and structural damping ratio is also studied, which points out that the sensitivity may limit the engineering application of nonlinear TMD. In the end of this paper, it is found that the nonlinear TMD is more applicable for determinate excitation, like wind, and it is also observed that the research in this paper emphasizes the simplified optimization strategy but not the applicability of the nonlinear TMD.

REFERENCES

1. Den Hartog, J. P. *Mechanical Vibrations*, McGraw-Hill, New York (1947).
2. Warburton G. B. "Optimum absorber parameters for various combinations of response and excitation parameters," *Earthq. Eng. Struct. Dyn.*, Vol. 10, pp. 381–491 (1982).
3. Setareh, M. and R. Hanson, "Tuned mass dampers to control floor vibration from humans," *J. Struct. Eng., ASCE*, Vol. 118, No. 3, pp. 741–762 (1992).
4. Xu, K. and T. Igusa, "Dynamic characteristics of multiple substructures with closed spaced frequencies," *Earthq. Eng. Struct. Dyn.*, Vol. 21, pp. 1059–1070 (1992).
5. Igusa, T. and K. Xu, "Vibration control using multiple tuned mass dampers," *J. Sound Vibr.*, Vol. 175, No. 4, pp. 491–503 (1994).
6. Roberson R. E. "Synthesis of a nonlinear dynamic vibration absorber," *J. Frankl. Inst.-Eng. Appl. Math.*, Vol. 254, pp. 205–220 (1952).
7. Carter, W. J. and F. C. Lin, "Steady-state behavior of nonlinear dynamic vibration absorber," *J. Appl. Mech.*, Vol. 28, No. 1, pp. 67–70 (1961).
8. Hunt, J. B. and J. C. Nrssen, "The broadband dynamic vibration absorber," *J. Sound Vibr.*, Vol. 83, No. 4, pp. 573–578 (1982).
9. Inaudi, J. A. and J. M. Kelly, "Mass damper using friction-dissipating devices," *J. Eng. Mech.*, Vol. 121, No. 1, pp. 142–149 (1995).
10. Abé M. "Tuned mass dampers for structures with bilinear hysteresis," *J. Eng. Mech.*, Vol. 122, No. 8, pp. 797–800 (1996).
11. Ricciardelli, F. and B. J. Vcikey, "Tuned vibration absorbers with dry friction damping," *Earthq. Eng. Struct. Dyn.*, Vol. 28, pp. 707–723 (1999).
12. Rüdinger F. "Optimal vibration absorber with nonlinear viscous power law damping and white noise excitation," *J. Eng. Mech. ASCE*, Vol. 132, No. 1, pp. 46–53 (2006).
13. Rüdinger F. "Tuned mass damper with nonlinear viscous damping," *J. Sound Vibr.*, Vol. 300, pp. 932–948 (2007).
14. Roberts, J. B. and P. D. Spanos, *Random Vibration and Statistical Linearization*, Wiley, Chichester (1990).
15. Crandall, S. H. and W. D. Mark, *Random vibration in mechanical systems*, Academic Press Inc, New York (1963).
16. Ayorinde, E. O. and G. B. Warburton, "Minimizing structural vibrations with absorbers," *Earthq. Eng. Struct. Dyn.*, Vol. 8, pp. 219–236 (1980).
17. Warburton G. B. "Optimum absorber parameters for minimizing vibration response," *Earthq. Eng. Struct. Dyn.*, Vol. 9, pp. 251–262 (1981).
18. Krenk, S. and J. Hogsberg, "Tuned mass absorbers on damped structures under random load," *Probab. Eng. Mech.*, Vol. 23, No. 4, pp. 408–415 (2008).
19. Krenk, S. and J. Hogsberg, *Tuned mass absorbers on damped structures*, 7th European Conference on Structural Dynamics, Southampton (2008).
20. Gradshteyn, I. S. and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th edition, Academic Press, New York (2007).

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